## Toynhee Curriculum Knowledge Maps

## MATHS [Shape]

## 2D Shapes



## Linked

 Knowledge Maps3D shapes / Transformations / Congruence and Similarity / Pythagoras and Trigonometry / Angles

## 3D SHAPE

## Keywords: Volume / Prism / Net / Face / Cross-section / Surface area / Pyramid

| Definition / Description: | Volume: The amount of space in a 3D container |  | : A 3D e with a rm -section | Net: A surface can be in a solid |  | Face: A flat surface of a solid shape | Cross-Section: A slice cut through at an angle $90^{\circ}$ to its axis | Surface Area: Total area of a solids exterior surface | Pyramid: A solid shape with triangular faces that meet at a vertex |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Knowledge points: | Nets |  | Plans elevation repres a 3D s | -2D <br> ations of <br> e | $\begin{aligned} & \text { Volur } \\ & \mathrm{V}=\mathrm{C} \\ & \mathrm{x} \text { Ler } \end{aligned}$ | me of prisms: Cross Section ngth | Surface area of prisms: <br> Total area of all faces | Volume of a <br> Pyramid - The volume of a pyramid is $\frac{1}{3}$ the volume of a prism. | Spheres: $V=\frac{4}{3} \pi r^{3}$ $S A=4 \pi r^{2}$ |
| Knowledge point examples: | Some shapes have more th possible net lik cube: | boid <br> ular rism <br> may <br> 1 <br> a | Front Elev <br> Side Elevatio $\square$ <br> lan view (B | n <br> seye) | Volu <br> =60 <br> $=36$ <br> 4 m <br> Volu <br> $8 \times 6$ <br> $=24$ <br> $=96$ | me $=$ <br> x 4 <br> $\times 4$ <br> $\mathrm{m}^{3}$ | Surface Area $=$ $\begin{aligned} & (6 \times 7)+(6 \times 7)+ \\ & (7 \times 12)+(7 \times 12) \\ & +(12 \times 6)+(12 \times 6) \\ & =2 \times(6 \times 7)+(7 \times 12) \\ & +(12 \times 6)=270 \mathrm{~m}^{2} \end{aligned}$ $\begin{aligned} & \pi \times 14^{2}=616 \\ & 2 \times \pi \times 14 \times 80 \\ & =7037 \end{aligned}$ | $\begin{aligned} & V=\frac{1}{3} \times 3 \times 3 \times 10 \\ & V=\frac{1}{3} \times 90 \\ & V=30 \mathrm{~m}^{3} \end{aligned}$ $\begin{aligned} & V=\frac{1}{3} \times \pi \times 4^{2} \times 10 \\ & V=\frac{1}{3} \times 160 \pi \\ & V=167.6 m^{3} \end{aligned}$ | Volume $=$ $\begin{aligned} & \frac{4}{3} \times \pi \times 5^{3} \\ & =\frac{500}{3} \pi \\ & \approx 523.6 \mathrm{~cm}^{3} \end{aligned}$ <br> Surface Area $=$ $\begin{aligned} & 4 \times \pi \times 5^{2} \\ & =100 \pi \\ & \approx 314.2 \mathrm{~cm}^{2} \end{aligned}$ |
| Linked <br> Knowledge <br> Maps | Pythagoras compound | Trig | ometry | mpou | and |  | $\begin{aligned} & 616+616+7037 \\ & =8700 \mathrm{~cm}^{2} \end{aligned}$ |  |  |

## Angles of polygons

## Keywords: Polygon / Regular / Interior / Exterior

Definition / Description:

## Knowledge points:

## Knowledge

 point examples:Polygon: A closed shape with all straight edges

## Sum of interior angles: $=(n-2) \times 180$

where n is the number of sides of the polygon
This formula is derived from the number of triangles that can be made in polygon from one vertex starting point and using the sum of interior angles of a triangle being $180^{\circ}$


| Shape | Sides | Sum of Interior Angles |
| :---: | :---: | :---: |
| , | 3 | 180 |
|  | 4 | 360 |
| $\square$ | 5 | 540 |
|  | 6 | 720 |
| 0 | 7 | 900 |
| A polygon with n sides |  | $(\mathrm{n}-2) \times 180$ |

Regular: When a polygon has equal sides and angles

Interior: The inside of a polygon


Exterior angle of a REGULAR polygon: = $360 \div$ n
where n is the number of sides of the polygon
What is the exterior angle of an octagon:
$360 \div 8=45^{\circ}$


A regular polygon has an exterior angle of $4^{\circ}$. How many sides does it have?

$$
\begin{aligned}
360 \div \mathrm{n} & =4 \\
\mathrm{n} & =360 \div 4 \\
\mathrm{n} & =90
\end{aligned}
$$

The regular polygon has 90 sides.

## Linked

 Knowledge Maps3D shapes / Transformations / Congruence and Similarity / Pythagoras and Trigonometry /
Angles

## ANGLES

## Keywords: $\quad$ Angle, Acute, Obtuse, Reflex, Right-Angle, Parallel

## Definition / Description:

Angle: A measure of turn

Acute: An angle that lies between $0^{\circ}$ and $90^{\circ}$

Obtuse: an angle that lies between $90^{\circ}$ and $180^{\circ}$

Reflex: an angle that lies between $180^{\circ}$ and $360^{\circ}$

## Right-Angle: a quarter of a revolution, or exactly $90^{\circ}$

Angle Facts: Recognise the properties of certain shapes and rules of angles

Angles in a triangle total $180^{\circ}$


Angles in a quadrilateral total $360^{\circ}$


Angles around a point total $360^{\circ}$


Angles on a straight line total $180^{\circ}$

$$
a^{0}+b^{0}=180^{\circ}
$$



Angles in Parallel Lines: Recognise the different classifications of equal angles within parallel lines

Alternate angles: when a line transects two parallel lines to create a " Z " or " S " shape, the inside angles are equal


Corresponding angles: when a line transects two parallel lines to create an " $F$ " shape, the angles on the parallel lines are equal


Vertically Opposite: when two lines intersect, angles opposite each other are equal


## Linked Knowledae

## Keywords:

## Definition / Description:

## Knowledge

 points:
## Knowledge point examples:

## Linked

 Knowledge MapsDiameter, Radius, Circumference, Chord, Arc, Sector, Segment, Tangent, Pi (m)

Diameter: the chord that passes through the centre of a circle
Radius: a line that joins the centre of a circle to the circumference
Circumference: The perimeter of a circle
Chord: a line that joins two points on the circumference
Arc: part of the circumference
Sector: the section of a circle between two radii and an arc
Segment: the section of a circle between a chord and an arc
Tangent: a straight line that touches a circle without crossing it
$\mathbf{P i}(\pi)$ : the ratio of a circumference to the diameter of a circle

Parts of a circle

$C=\pi d$

$$
=\pi \times 6
$$

$$
=18.8 \mathrm{~cm}
$$

(1dp)



Perimeter of a sector

When calculating the perimeter of
a sector we first calculate the arc
a sector we first calculate the arc length and then add on 2 radii radii
is the plural word for radius). is the ellural word for radius).
Usually measured in $\mathrm{cm}, \mathrm{m}, \mathrm{mm}$.

Area of sector: Length of arc: $A=\frac{\theta}{360} \times \pi r^{2} \quad L=\frac{\theta}{360} \times \pi d$

Where $\Theta$ is the angle
Where $\theta$ is the angle


Angles
Circle Theorems
Non-linear graphs - circle, reciprocal, exponential 3D shapes

| Keywords: | Compound / Density / Pressure / Newton |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Definition / Description: | Compound: A Mixture | Density: an objects mas per unit volume | Pressure: Force per unit area | Newton: Unit for weight and force |
| Knowledge points: | Speed Distance, Time: <br> Speed = Distance $\div$ Time | Average Speed: <br> Total Distance $\div$ Total Time | Density: <br> Density $=$ Mass $\div$ Volume | Pressure: <br> Pressure = Force $\div$ Area |
| Knowledge point examples: | If I travel 72 miles in 3 hours what is my speed? <br> Speed = Distance $\div$ Time <br> 72 miles $\div 3$ hours $=\underline{\mathbf{2 4 m p h}}$ <br> Mark cycles 42 km at an average speed of $14 \mathrm{~km} / \mathrm{h}$. How long does it take him? <br> Time $=$ distance $\div$ Speed <br> $42 \mathrm{~km} \div 14 \mathrm{~km} / \mathrm{h}=\underline{3 \text { hours }}$ <br> A bird flies for 40 minutes at an average speed of $11 \mathrm{~m} / \mathrm{s}$. How far does the bird fly in kilometres? <br> 40 minutes $=2400$ seconds Distance $=$ Speed $x$ Time $11 \mathrm{~m} / \mathrm{s} \times 2400 \mathrm{~s}=26400 \mathrm{~m}$ $=\mathbf{2 6 . 4} \mathrm{km}$ | A car travels 60 km at $30 \mathrm{~km} / \mathrm{h}$ and then a further 180 km at 160 km/h. Find: <br> a) the total time taken in hours: $\begin{aligned} & \text { Time }=\text { distance } \div \text { Speed }= \\ & =60 \div 30=2 \text { hours } \\ & =180 \div 160=1.125 \text { hours } \\ & =\mathbf{3 . 1 2 5} \mathbf{h r s} \end{aligned}$ <br> b) the average speed for the whole journey $=(60+180) \div 3.125$ $=76.8 \mathrm{~km} / \mathrm{h}$ | A piece of silver has a mass of 42 g and a volume of $4 \mathrm{~cm}^{3}$. Work out the density of silver Density $=$ Mass $\div$ Volume $=42 \mathrm{~g} \div 4 \mathrm{~cm}^{3}=10.5 \mathrm{~g} / \mathrm{cm}^{3}$ <br> A 50 g piece of wood which has a density of $0.4 \mathrm{~g} / \mathrm{cm}^{3}$ Work out the volume of the block. <br> Volume $=$ Mass $\div$ Density $50 \mathrm{~g} \div 0.4 \mathrm{~g} / \mathrm{cm}^{3}=125 \mathrm{~cm}^{3}$ | A force of 30 Newtons is applied to an area of $1.5 \mathrm{~m}^{2}$. Work out the pressure in $\mathrm{N} / \mathrm{m}^{2}$ <br> Pressure $=$ Force $\div$ Area $30 \mathrm{~N} \div 1.5 \mathrm{~m}^{2}=20 \mathrm{~N} / \mathrm{m}^{2}$ <br> A force is applied to an area of $4.5 \mathrm{~m}^{2}$. <br> It produces pressure of 12 $\mathrm{N} / \mathrm{m}^{2}$. <br> Work out the force in Newtons. <br> Force $=$ Pressure $\times$ Area $12 \mathrm{~N} / \mathrm{m}^{2} \times 4.5 \mathrm{~m}^{2}=\underline{54 \mathrm{~N}}$ |
| Linked Knowledge Maps | Non-compound measures / Bounds |  |  |  |

Keywords:

## Scale factor / Ratio / Enlargement / Similar / Congruent / Identical /

| Definition / | Scale factor: The <br> ratio of the <br> enlarged distance <br> to the original <br> Description: | Ratio: A part to <br> part comparison | Enlargement: <br> Changing the size <br> of a shape by a <br> given scale factor | Similar: Two <br> shapes whose <br> sides are in <br> proportion to one <br> another | Congruent: How <br> to mathematically <br> describe 2 shapes <br> that are identical | Identical: Exactly |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| alike |  |  |  |  |  |  |

## Knowledge points: <br> Use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)

## Knowledge point examples:

Understand and identify congruent triangles; prove congruency using formal arguments


Condition for Congruency = Angle, Side, Angle (ASA)

Condition for Congruency = Side, Angle, Side (SAS)


Condition for
Congruency = Right Angle, Hypotenuse, Side (RHS)

## Use congruence and

 similarity to prove missing angles and sidesRecognise similar shapes when rotated or reflected; apply mathematical reasoning
$A B C$ is a straight line. Work out $x$.


Triangle $A B$ and Triangle $B C$ are both isosceles triangles $>$ Angles at $A$ and $B$ are equal and angles at $C$ and corresponding base are equal.
Angle at $B$ in Triangle BC: $180^{\circ}-\left(40^{\circ}+40^{\circ}\right)=100^{\circ}$ Angles on a straight line total $180^{\circ}$, therefore angle at $B$ in Triangle $A B=80^{\circ}$. Angles at $A$ and $B$ are equal, so $x$ is $180^{\circ}-\left(80^{\circ}+80^{\circ}\right)=$ $20^{\circ}$.

Compare lengths, areas and volumes using ratio notation
Make links to similarity and scale factors


Write the ratio perimeter $A$ : perimeter $B$ in its simplest form.

Perimeter A:
$2(7+2)=18 \mathrm{~cm}$
Perimeter B:
$4+4+4 \mathrm{~cm}+4$
$=16 \mathrm{~cm}$
$\left.\begin{array}{r}\text { Ratio }=18: 16 \\ \underline{9: 8}\end{array}\right) \div 2$
Write the ratio area $A$ : area
$B$ in its simplest form.
Area $A: 7 \times 2=14 \mathrm{~cm}^{2}$
Area B: $4 \times 4=16 \mathrm{~cm}^{2}$
Ratio $=14: 16) \div 2$

Apply the concepts of congruence and similarity, including relationships between lengths, areas and volumes

These boxes are similar.


What is the ratio of the volume of box $A$ to the volume of box $B$ ?

Ratios of side lengths = $2 \mathrm{~cm}: 6 \mathrm{~cm}=1: 3$ (in simplest form)

If length ratio is $a: b$, then area ratio is $a^{2}: b^{2}$ and volume ratio is $a^{3}: b^{3}$.

Therefore, ratio of volumes = $1^{3}: 3^{3}=1: 27$

## MEASURES

## Keywords: Metric / Capacity / Mass / Imperial

## Definition / Metric: A number system

 Description:based on multiples of 10

Capacity: How much liquid a 3D solid can hold

Mass: How heavy an object is

Imperial: a system of weights and measures that includes pounds, ounces, feet, yards, miles, etc

## Knowledge

 points:
## Knowledge

## point

 examples:| 1 centimetre | $=10$ millimetres |  |
| :--- | :--- | :--- |
| 1 metre | $=100$ centimetres |  |
| 1 kilometre | $=1000$ metres |  |
| 1 cm | $=$ | 10 mm |
| 1 m | $=100 \mathrm{~cm}$ |  |
| 1 km | $=1000 \mathrm{~m}$ |  |

$$
7 \mathrm{~cm}=70 \mathrm{~mm}
$$

Metric Units of length:

$$
90 \mathrm{~mm}=9 \quad \mathrm{~cm}
$$

$$
350 \mathrm{~cm}=3.5 \mathrm{~m}
$$

## Metric Capacity and Mass

Imperial Units

| Metric Capacity |  |
| :---: | :---: |
| 1000 mL | 1 L |

$\underline{3} L=3000 \mathrm{~mL}$
$8 \mathrm{~L}=\underline{8,000} \mathrm{~mL}$

| Metric Mass |  |
| :---: | :---: |
| 1000 mg | 1 g |
| 1000 g | 1 kg |

$43 \mathrm{~g}=0.043 \mathrm{~kg}$
$57 \mathrm{~g}=0.057 \mathrm{mg}$

Length
1 inch $=2.5 \mathrm{~cm}$
$1 \mathrm{foot}=30 \mathrm{~cm}$
1 yard $=90 \mathrm{~cm}$ 5 miles $=8 \mathrm{~km}$

Capacity
1.75 pints $=1$ litre $9 \mathrm{fl} \mathrm{oz}=250 \mathrm{ml}$

Weight
8 ounces $=225 \mathrm{~g}$
2.2 pounds $=1 \mathrm{~kg}$

Area and Volume conversion


Linked Place Value Decimals Rounding, Estimation Bounds / FDP conversion / Compound measures

## PYTHAGORAS AND TRIGONOMETRY

| Keywords: | Hypotenuse / Opposite / Adjacent / Complementary angle / Square Root / Inverse |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Definition / Description: | Hypotenuse: The longest side of a right angled triangle | Opposite: The side opposite the given angle | Adjacen side in be the given and the rig angle | :The tween angle ight | Complementary <br> : Angles to add up to $90^{\circ}$ | Square Root: A number which produces a specified quantity when multiplied by itself. | Inverse: The reverse or opposite |
| Knowledge points: | Calculate missing sides of a right angled triangle <br> Use Pythagoras to solve problems in 3D <br> Work fluently with the hypotenuse, opposite and adjacent sides Use the tangent, sine and cosine ratio to find missing side lengths Use sine, cosine and tangent to find missing angles Select the appropriate method to solve right-angled triangle problems |  |  |  |  |  |  |
| Knowledge point examples: | Finding the hypotenuse: $\begin{aligned} & a^{2}+b^{2}=c^{2} \\ & 5^{2}+12^{2}=x^{2} \\ & 25+144=x^{2} \\ & 169=x^{2} \\ & \sqrt{169}=x \\ & x=13 \mathrm{~cm} \\ & x \mathrm{~cm} \quad 12 \mathrm{~cm} \end{aligned}$ | Finding the S Side | orter $\begin{aligned} & 2-b^{2}=a^{2} \\ & 2-8^{2}=x^{2} \\ &-64=x^{2} \\ & 36=x^{2} \\ & \sqrt{36}=x \\ & x=6 \mathrm{~cm} \end{aligned}$ | 3D Py $\begin{aligned} & \mathrm{AG}=1 \\ & =\sqrt{7^{2}} \\ & =\sqrt{76} \\ & =8.6 \end{aligned}$ | $\frac{a^{2}+b^{2}+c^{2}}{+3^{2}+4^{2}}$ | SOHCAHTOA: Side <br> Label your triangle and select the correop $\ddagger p$ tio $\tan \theta=\frac{}{a d j}$ $\tan 40=\frac{x}{5}$ $\begin{array}{r} 5 \times \tan 40=x \\ 4.19 \mathrm{~cm}=x \end{array}$ | SOHCAHTOA : Angle <br> Label, select ratio and Do not forget to use sin-1 when finding the angle <br> Let angle $\mathrm{ABC}=\theta$ $\begin{gathered} \sin \theta=\frac{6}{10} \\ \theta=\sin ^{-1} \frac{6}{10} \\ \begin{array}{c} 8687^{\circ}(20 \mathrm{dp}) \end{array} \\ 6 \mathrm{~cm} \quad 8 \mathrm{~cm} \end{gathered}$ |

## Linked <br> Knowledge <br> Maps:

## Keywords:

## Definition /

 Description:
## Knowledge

 points:
## Knowledge point

## Linked

 Knowledge MapsTranslation / Vector / Rotation / Reflection / Symmetry / Enlargement
Translation: When a shape is moved into a different position without being turned or flipped

Translation:

- Column Vector

Vector: The description of a movement for a translation

Rotation:

- Centre of Rotation (x,y)
- Direction (clockwise/anticlockwise)
- Angle of Rotation

Rotation: The circular motion of an object are a centre

Reflection:

- Mirror Line
(equation of straight line)

A fractional scale factor generates a SMALLER image.
Reflection: When a shape is reflected in a mirror line it is flipped

Enlargement:

- Centre of Enlargement (x,y)
- Scale Factor

Symmetry : A mirror image

Enlargement: When a shape changes size

Enlargement - negative scale factor

When the scale factor is negative the enlarged shape appears on the other side of the centre of enlargement



2D shapes, Congruence and Similarity, Linear Graphs, Vectors, Scale



Enlarge shape A by scale factor $\frac{1}{2}$ about the point.


## Definition /

 Description:
## Knowledge

 points:
## Knowledge

 point examples:Opposite: The side opposite the given angle

Label triangle to use with trigonometric formulae

Adjacent: The side in between the given angle and the right angle

Perpendicular:
Two sides that are at a right angle to one another

Inverse: To apply an opposite function

Subject: The
unknown variable of a formula

Know and apply the cosine rule to find unknown angles and sides $a^{2}=b^{2}+c^{2}-2 b c \cos \mathrm{~A}$

Use the cosine rule to work out the unknown


Area $=\frac{1}{2} a b \sin C$
Area $=\frac{1}{2} \times 3 \times 8 \times \sin 38$
Area $=7.4 \mathrm{~cm}^{2}$

Area of the triangle $=1.5 \mathrm{~m}^{2}$
Calculate the size of angle $\theta$.


Area $=\frac{1}{2} a b \sin C$
$1.5=\frac{1}{2} \times 1.7 \times 2.8 \sin \theta$
$1.5=2.38 \sin \theta$
$\frac{1.5}{2.38}=\sin \theta$
$\theta=\sin ^{-1}\left(\frac{1.5}{2.38}\right)$
$\theta=39.1^{\circ}$

## Linked

## VECTORS

Keywords: Vector / Magnitude / Parallel / Scalar

Definition / Description:

Vector: a quantity with size and direction

Magnitude: The size of a vector (length)

Parallel: Two lines that never meet

Scalar: A quantity that has only magnitude

## Knowledge points:

## Knowledge

 point examples:
$\binom{-9}{15}$

Move 9 places
to the LEFT and 15 places UP

## Parallel vectors: <br> Vectors that are multiples of one subtracting vectors:

 another. If one vector is parallel to another but a different size, they are SCALAR MULTIPLEs$\mathbf{a}=\binom{2}{4} \mathbf{b}=\binom{-2}{-4}$ Vectors $\mathbf{a}$ and $\mathbf{b}$ are parallel and scalar multiples (multiple of -1 )
$\mathbf{c}=\binom{4}{8} \mathbf{b}=\binom{3}{6}$
Vectors $\mathbf{c}$ and $\mathbf{d}$ are parallel and scalar multiples (multiple of -1 )

Adding and Multiplying Adding vectors is equivalent to applying one vector followed by the other

$$
\mathbf{a}=\binom{5}{3} \mathbf{b}=\binom{3}{-2}
$$

$$
\mathbf{p}=\binom{2}{-3}
$$

$\mathbf{a}+\mathbf{b}=\binom{5+2}{3+-2}$

$$
2 p=\binom{2 \times 2}{2 \times-3}=\binom{4}{-6}
$$

$$
-\mathbf{p}=\binom{-1 \times 2}{-1 \times-3}=\binom{-\mathbf{2}}{\mathbf{3}}
$$

Vector geometry:

a) $\overrightarrow{\mathrm{AO}}=-\mathbf{a}$
b) $\overrightarrow{\mathrm{AB}}=-\mathbf{a}+\mathbf{b}$
c) $\overrightarrow{\mathrm{BA}}=-\mathbf{b}+\mathbf{a}$

