Toynbee Curriculum Knowledge Maps

MATHS (Algebra)

Toynbee School



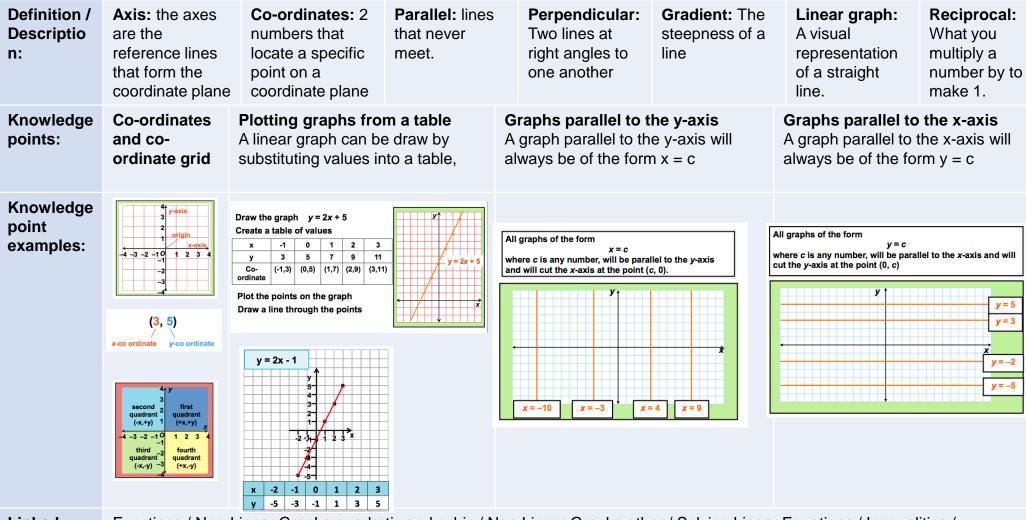
| | | | Sequences | • | | | | | | |
|---------------------------------|---|--|---|---|---|---|--|--|--|--|
| Keywords: | Arithmetic / <i>n</i> th term / | Arithmetic / <i>n</i> th term / Geometric / Term / Quadratic / Iterate | | | | | | | | |
| Definition / Description: | Arithmetic – a sequence where terms are found by adding or subtracting an equal amount. | Nth term – The general rule of a number sequence. | Geometric - A sequence in which you find each term by multiplying the previous term by a fixed value. | Term – a part of an equation, expression or sequence. | | Quadratic – A sequence where the difference increases or decrease by an equal amount each time | Iterate - a quantity arrived at by iteration. | | | |
| Knowledge points: | Nth term of a linear sequence | Finding terms in a sequence | a Nth term of a Qu sequence | uadratic | Geome | tric Progression | Sequences by iteration | | | |
| Knowledge point examples: | n: 1 2 3 4 $2 \underbrace{5}_{+3} \underbrace{8}_{+3} \underbrace{11}_{3n-1}$ The nth term of a linear sequence is always of the form An \pm b, where: A, is the difference between each term and the next term. b is the difference between the first term and A. n: 0 1 2 3 $13 \underbrace{11}_{+2} \underbrace{9}_{-2} \underbrace{-2}_{-2} \underbrace{-2}_{-2}$ In a descending sequence we find the <u>zero term</u> to discover what we are taking An way from. | work out the 50 th term. The nth term of this sequence is 7n – 2 Find the 50th terr by substituting | Term 5 9 1 st Diff. 2^{nd} Diff. 2^{nd} Diff. 2 The second different constant (2) so the quadratic and the c | 15 	 23 6 	 2 	 2 The constraints of the sequence is coefficient of the constraints of the constraint | is a sec zero nu each te is found the prev number Find the terms of $3 \begin{array}{c} 6 \\ x^2 \\ x^2 \end{array}$ The term here is x | $\frac{2}{2} \underbrace{24}_{2 \times 2}$ n to term rule (2 therefore the (2 therefore the (2 therefore the (3 therefore the (4 the terms are) (4 the term | Find the first four iterations of the iterative formula $x_{n+1} = 3x_n - 2$ with $x_1 = 2$. $x_2 = 3x_1 - 2$ $= 3 \times 2 - 2 = 4$ $x_3 = 3x_2 - 2$ $= 3 \times 4 - 2 = 10$ $x_4 = 3x_3 - 2$ $= 3 \times 10 - 2 = 28$ $x_5 = 3x_4 - 2$ $= 3 \times 28 - 2 = 82$ | | | |
| Linked Knowledge | Notation and manipul | lation / Functions / Mu | ltiples, Primes, Facto | rs / Index Nu | mbers | | | | | |

| | | AL | GE | BRAIC N | OTATION | AND | MAI | NIPULATI | ON | | | |
|---------------------------------|---|--|---|---|--|-------------------------------|--|---|---|----------------------------|-----------------------------------|--|
| Keywords: | Expression / Si | Expression / Simplify / Term / Variable / Substitute / Coefficient / Equivalent / Solve / Expand / Factorise | | | | | | | | | | |
| Definition / Description: | Expression: an algebraic statement | Simplify collect li terms | | Term: part of an expression | Variable: a quantity that can have different values | Substit exchar or repla | nge | Coefficient: a number or letter multiplying a term | Solve: find the result | Expar multip of brac | ly out | Factorise: separate into factor: |
| Knowledge points: | Use and interp notation Use letter symbol represent unkn numbers in equ | ools to own | form Swa sym form num solv | stitute into nulae p letter bols in nulae for bers to e for an nown | Understand and use expressions, (equations, formulae, identities, inequalities and terms f | | Simplify expressions Collect like terms, expanding brackets, factorising into brackets | | Rearrange formulae Balance terms about the equation sign to make another unknown the subject of the formula | | identi Use a prove expre | algebra to e equivalent essions and ne identity |
| Knowledge point examples: | <i>ab</i> in place of <i>3y</i> in place of <i>y</i> and 3 x a^2 in place of <i>a</i> in place of <i>a</i> $\frac{a}{b}$ in place of | γ+y+y y axa,a³ ∢axa | of x ² Whe repla x ² w | the value when $x = 5$ en $x = 5$, ace the x in with 5 to the $(5)^2$. 25 | Expression: a Equation: $b = a$ Formula: $F = n$ Identity: $2 + b \equiv$ Inequality: $a >$ Term: a or $2b$ | a + 1 na : b+2 | = a + 3a - | b + 2a - 2b + 2a + b - 2b = b and: + 5) x x) + (3 x 5) x + 15 torise: 8d x 3c - 2 x 4d | Rearrange: + 3 to make subject y = 2x + 3 (-3 on both s y - 3 = 2x (÷2 on both sides) $\frac{y - 3}{2} = x$ | x the sides) | 2(a + | x b ≡ b ³ b) ≡ 2a + 2 6x ≡ 11x |
| Linked Knowledge | Functions (Incl | . Compos | ite / I | nverse) / Inequ | ualities / Solve L | inear Eq | Juatior | ns / Solve Quad | Iratic Equation | S | | |

Maps

AXES AND CO-ORDINATES

Keywords: Axis / Co-ordinate / Parallel / perpendicular / Gradient / Linear Graph / Reciprocal



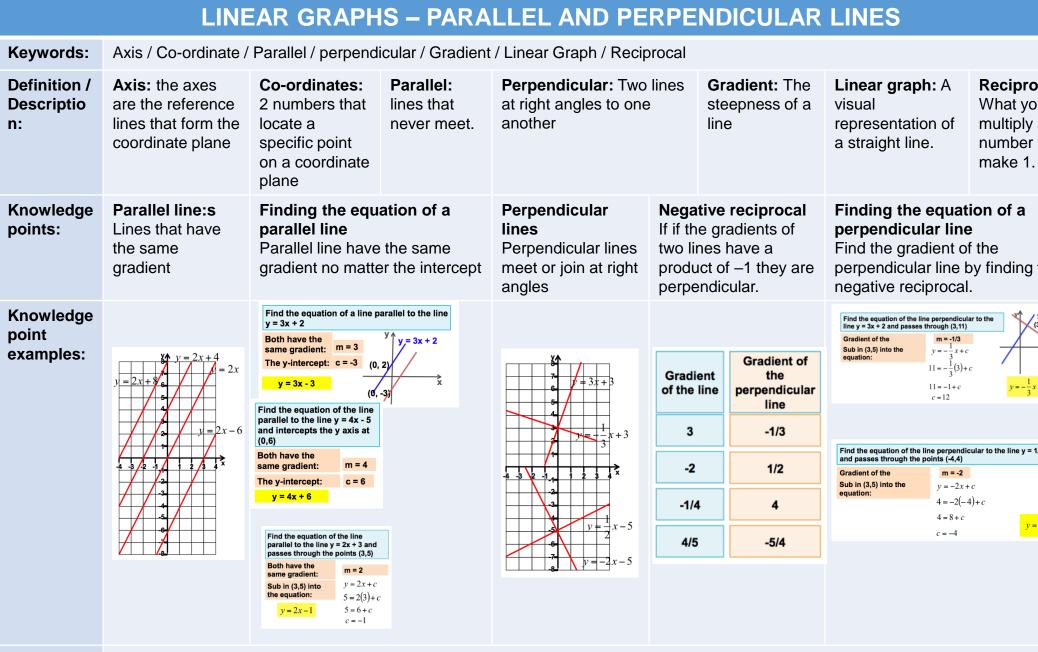
Linked Functions / Non-Linear Graphs quadratic and cubic / Non-Linear Graphs other / Solving Linear Equations / Inequalities / Simultaneous equations / Sequences / Transformations / Linear Graphs / Linear Graphs – parallel and perpendicular lines Maps

Functions

| Keywords: | Functions, input, output, inverse function, composite function, flow charts | | | | | | | | |
|---------------------------------|---|--|--|---|--|--|--|--|--|
| Definition / Description: | A function is a relationship between variables. The inverse function is the reverse process. A composite function is the succession of two functions. | | | | | | | | |
| Knowledge points: | Interpret simple functions as expressions with inputs and outputs | Understand and use the function | Solve equations that use the function notation | Interpret and use the inverse function | Interpret and use the composite function | | | | |
| Knowledge point examples: | Input $x = -8$ Write down the output y as an expression in terms of x . y = 5x - 8 Input $-4 + 2 + y$ Work out the output when the input is 10 $(10 - 4) \div 2 = 3$ | Given that $f(x) = 4x - 5$ work out (a) $f(-6)$ (b) $f(0.5)$ (a) $4 \times -6 - 5 = -29$ (b) $4 \times 0.5 - 5 = -3$ | f(x) = 3x + 2 Solve $f(x) = 0$ 3x + 2 = 0 3x = -2 $x = -\frac{2}{3}$ | $g(x) = \frac{7x-1}{2}$ Find $g^{-1}(x)$ $y = \frac{7x-1}{2}$ 2y = 7x - 1 2y + 1 = 7x $x = \frac{2y + 1}{7}$ $g^{-1}(x) = \frac{2x + 1}{7}$ | $f(x) = 5x + 1, g(x) = x^{2}$ $fg(x) = f(x^{2})$ $= 5 x^{2} + 1$ | | | | |
| Linked Knowledge Maps | Solving linear equations | | | | | | | | |

| INEQUALITIES | | | | | | | | | | |
|---------------------------------|--|--|--|---|---|--|--|--|--|--|
| Keywords: | Inequality, region, solve, e | Inequality, region, solve, equation, variable, linear, quadratic | | | | | | | | |
| Definition / Description: | An inequality is a statement showing two quantities that are not equal. They can be represented on a number line and on a graph. | | | | | | | | | |
| Knowledge points: | Inequality notation Know correct conventions of open circle for strict inequality and closed circle for inclusive inequality | Represent Inequalities on a number line Show inequalities on a number line using correct notation | Solving linear inequalities Solve inequalities in one and represent solution set on a number line and using set notation. | Graphical Inequalities Represent inequalities on a coordinate grid | Solve Quadratic inequalities Solve quadratics and represent answers on a number line and on a graph | | | | | |
| Knowledge point examples: | x > 1 x is greater than 1 x < 5 x is less than 5 $x \ge 2$ x is greater or equal to 2 $x \le 0$ x is less than or equal to 0 $-3 \le x < 5$ X is greater or equal to negative 3, and smaller than 5 | When we represent (plot) inequalities, we must show whether they include or exclude the starting number. • includes • excludes $x \ge 2$ $4 -3 -2 -1 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6$ $-3 \le x < 5$ $4 -3 -2 -1 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6$ | $5y + 10 \le 40$ $-10 -10$ $5y \le 30$ $\div 5 \div 5$ $y \le 6$ $3 < 2x - 7 \le 9$ $+7 +7$ $10 < 2x \le 16$ $\div 2 \div 2$ $5 < x \le 8$ $\{6, 7, 8\}$ | Where on the graph is $y \ge x$? y = x y | $x^2 \le 9$ Form & solve an equation to find the two bounds. $x^2 = 9$ x = 3 or x = -3 $-3 \le x \le 3$ Solve the inequality $x^2 + 3x - 4 < 0$ 1. Factorise 2. Set y = 0 3. Sketch function 4. It is < 0 so we shade in under the x axis. $y = -4 \le x \le 1$ | | | | | |
| Linked Knowledge Maps | Solving linear equations Solving quadratic equations Linear graphs Non linear graphs including qua | udratic | | | -4 < x < 1 | | | | | |

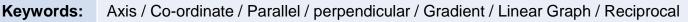
| | SOLVING EQUATIONS | | | | | | | | |
|---------------------------------|--|---|---|--|--|----------|---|--|--|
| Keywords: | Solve / Equation / Coefficient / Inverse / Equal / Linear Equation | | | | | | | | |
| Definition / Description: | Solve : To find the answer/value of something | Equation : A mathematical statement that shows two things are equal | Coeffic number multiply algebrai | r or letter reverse or ving an opposite | | | Equal : An equals sign shows the equality between two expressions | Linear Equation : An equation where no variable has a power greater than one | |
| Knowledge points: | Solve simple linear where the equation | Solve simple linear equations by using inverse operations Solve simple linear equations with integer coefficients where the unknown appears on one or both sides of the equation or where the equation involves brackets Set up a simple linear equation to solve problems | | | | | | | |
| Knowledge point examples: | One Step 3a = 15 $(\div 3)$ $(\div 3)$ a = 5 | 4a = 20 | $\begin{array}{c} p & With \\ 17 & 2(a + 1) \\ (+3) & 2a + 1 \\ 0 & (-6) \\ (\div 4) & 2a \\ (\div 2) & 2$ | | (-3) = 11 (-6) = 6 = 11 (-3a) | | nknowns on both sides + 7 = 5a + 11) (-3a) 7 = 2a + 11 1) (-11) -4 = 2a 2) (÷2) a = -2 | | |
| Linked Knowledge Maps: | Algebraic Manipulat sequences | tion and Notation / L | inear Gra | aphs / quad | ratic equations | ; / inec | qualities / simultaneo | us equations / linear | |

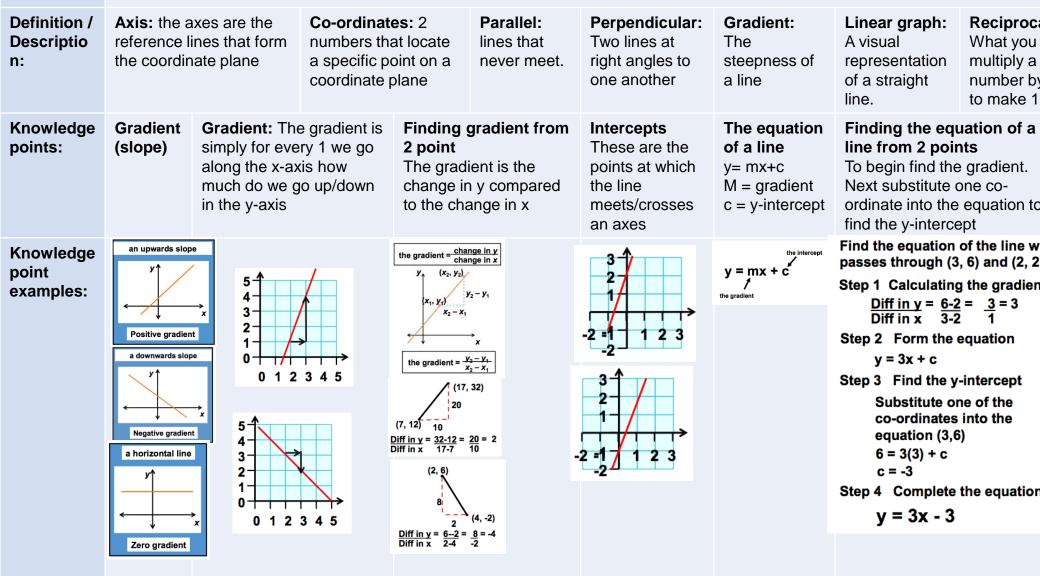


Linked Knowledge Maps

Functions / Non-Linear Graphs quadratic and cubic / Non-Linear Graphs other / Solving Linear Equations / Inequalities / Simultanec equations / Sequences / Transformations / Linear Graphs/ Axes and Coordinates

LINEAR GRAPHS





Linked Knowledge Maps Functions / Non-Linear Graphs quadratic and cubic / Non-Linear Graphs other / Solving Linear Equations / Inequalities / Simultanec equations / Sequences / Transformations Linear Graphs – parallel and perpendicular lines / Axes and Coordinates

Non linear graphs

| | - | | | | | |
|---------------------------------|---|---|--|---|--|--|
| Keywords: | Reciprocal / Asym | nptote / Exponential / G | Frowth and decay / Rad | lius / Non-Linear | | |
| Definition / Description: | Reciprocal: The inverse of any number except 0 | Asymptote: the distance between a curve and a line which approaches but never touches zero | Exponential: a function, where we use repeated multiplication on an initial value to get the output | Growth and decay: an example of exponential increase (growth) and decrease (decay) | Radius: The distance between the centre of a circle and it's circumference | Non-Linear: A graph which does not have a consistent gradient |
| Knowledge points: | Reciprocal graph: asymptote to the divide by 0 | remember the curve as we cannot | Exponential Graph: A in the form $y = a^x$ will the point (0,1) | | Equation of a circle: T circle with the centre (the form: $x^2 + y^2 = r^2$ | - |
| Knowledge point examples: | | | Use the table to $y = \frac{4}{\frac{1}{4}}$ | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | | ations of the circles. |
| Linked Knowledge | Non-Linear Graph | ns quadratic and cubic / | Linear Graphs / Funct | ions / | | |

Knowledge Maps

Non-linear Graphs – quadratic and cubic

| Keywords: | Quadratic / Parabola / Subs | stitute / Cubic / Root / Solution | | | | |
|---------------------------------|--|---|---|--|---|--|
| Definition / Description: | Quadratic expression contains terms up to and including x ² | A Parabola is a curved graph formed from a quadratic equation. A parabola has a line of symmetry | ph formed from a adratic equation. A abola has a line ofterms up to and including x³ with amountsExchange or replace variables with amounts | | Roots or solutions: of graphs are the values of x-coordinates of the point where the graph crosses the x-axis | |
| Knowledge points: | | ues for a graph, substituting di nooth line to create a curve | fferent values of x to find the y c | oordinate | | |
| Knowledge point examples: | Positive Quadratic in the form $y = x^2$ | Negative quadratic in the form $y = -x^2$ | Positive cubic graph y = x ³ | Negative cubic graph y = -x ³ y y y -2 -1 0 1 2 y y -2 -1 0 y y y y y y y y y y y y y | Complex cubic graph $(0,3)$ y $y = x^3 - 3x^2 - x + (1,0)$ $(-1,0)$ $(3,0)$ | |
| Linked Knowledge Maps | Linear Graphs Non-linear graphs – reci Solving Quadratic equat | procal, exponential and cir ions | cle | | | |

| Solving Quadratic Equations | | | | | | | | | | | |
|----------------------------------|--|--|--|--|--|--|--|--|--|--|--|
| Keywords: | Quadratic Equation / Sol | Quadratic Equation / Solution / Formula / Factorise / Discriminant | | | | | | | | | |
| Definition / Descriptio n: | Quadratic Equation: An equation where the maximum power is two | Solution: The answer to a quadratic equation | Formula: An equation to fund quantities when gir certain values | • | Discriminant: the part of the quadratic formula underneath the square root symbol | | | | | | |
| Knowledge points: | Solving by factorising – Use the product and sum | Solving when not equal to zero | Solving by completing the square | Solve using the quadratic formula – Use thew formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ | Finding the amount of solutions a quadratic equation has. The value of the discriminant shows the amount of solutions a quadratic equation has | | | | | | |
| Knowledge point examples: | Factorise and solve $g^2 + 11g + 24 = 0$ Factors of 24 1 and 24 2 and 12 3 and 8 4 and 6 $g^2 + 11g + 24 \equiv$ (g + 3)(g + 8) (g + 3)(g + 8) = 0 g = -3 and $g = -8$ | A quadratic equation can only be solved when equal to zero Factorise and solve: $x^{2} + 6x + 10 = 2$ $-2 - 2$ $x^{2} + 6x + 8 = 0$ $(x+4)(x+2) = 0$ $x = -4 \text{ or } -2$ | Use $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$ To complete the square before solving: $x^2 + 8x + 6 = 0$ $(x + 4)^2 - 10 = 0$ $(x + 4)^2 = 10$ $(x + 4)^2 = 10$ $(x + 4) = \pm\sqrt{10}$ $x = \pm\sqrt{10} - 4$ | Solve $3x^2 + 8x - 5 = 0$ using the quadratic formula To 3 S.F $a = 3 b = 8 c = (-5)$ $x = \frac{-8 \pm \sqrt{(64 - 4 \times 3 \times -5)}}{2 \times 3}$ $x = \frac{-8 \pm \sqrt{124}}{6}$ x = 0.523 or $x = -3.19$ | $b^{2} - 4ac > 0$ 2 solutions $3x^{2} - 4x - 3 = 0$ $b^{2} - 4ac =$ $(-4)^{2} - 4 \times 3 \times (-3) = 52$ <u>Two Solutions</u> $b^{2} - 4ac = 0$ 1 solution $16x^{2} + 16x + 4 = 0$ $b^{2} - 4ac =$ $16^{2} - 4 \times 16 \times 4 = 0$ <u>One solution</u> $b^{2} - 4ac < 0$ No real solutions $4x^{2} + 3x + 2$ $b^{2} - 4ac =$ $3^{2} - 4 \times 4 \times 2 = -23$ <u>No real solutions</u> | | | | | | |

Linked Multiples, Primes, Factors / Notation and manipulation / Non-Linear Graphs quadratic and cubic / Solving Linear Equations / Inequalities / Sequences / Simultaneous equations Maps

| | | SIMULTANE | ous | EQUATION | S | | | | |
|---------------------------------|--|---|---|--|-------------------|---|---|--|--|
| Keywords: | Simultaneous / Eliminate / Non-linear / Linear / Substitute | | | | | | | | |
| Definition / Description: | Simultaneous: 2 values Elimit satisfying more than one equation at the same time | nate: To remove | or equation where there is a equat | | | : An expression or on where the only than 1 | Substitute: To exchange or replace a value | | |
| Knowledge points: | Solving using Elimination (2 Linear) Label equations Look to see if the coefficients of either variable are equal Multiply one or both equations to make the coefficients of one variable the same Look at sign in front of those variables Same Signs Subtract Different Signs Add Solve equation to find first variable Substitute into original equation to find second variable Check both solutions in other equation | Solving using Substitut (1 linear and 1 quadra Rearrange one equation to make variable as subject Substitute into sec equation Solve as linear equation Substitute into orig equation to find of variable Check both solution the other equation | atic) a ct cond ginal ther ons in | Simultaneous Equa Graphically 1. Plot both graph 2. The solution to simultaneous equation is four the intersection two graphs | s the nd at | Setting up and solving: David and Jen are attending a football match with their families Jen buys 2 Adult tickets and 2 Kids tickets for £90 David buys 1 Adult ticket and 3 Kids tickets for £75 Use Simultaneous Equations to work out the price of 1 Adult and 1 Kids Ticket. | Solving using Substitution (1 linear and 1 quadratic) 1. Rearrange linear to make variable as subject 2. Substitute into quadratic 3. Solve quadratic (2 solutions) 4. Substitute both solutions in the linear equation to find other variable | | |
| Knowledge point examples: | 7x + 6y = 46 (1) 2x + 3y = 17 (2) (2) x 2 4x + 6y = 34 (3) SSS (1)-(3) 3x = 12 x = 4 Sub in (1) 7(4) + 6y = 46 28 + 6y = 46 6y = 18 y = 3 Check in (2) 2(4) + 3(3) = 17 | 3x + 2y = 21 y = x + 3 3x + 2(x + 3) = 21 3x + 2x + 6 = 21 5x + 6 = 21 5x = 15 x = 3 y = 3 + 3 y = 6 3(3) + 2(6) = 21 | | x = 6, y = 7 | = 32 | 2a + 2k = 90 (1) a + 3k = 75 (2) (2) X 2 2a + 6k = 150 (3) SSS (3) - (1) 4k = 60 k = 15 a + 3(15) = 75 a = 30 Adults = £30 Kids = £15 | $x^{2} - y^{2} = 7$ $2y = 2 + x$ $x = 2y - 2$ $(2y - 2)^{2} - y^{2} = 7$ $4y^{2} - 8y + 4 - y^{2} = 7$ $3y^{2} - 8y - 3 = 0$ $(3y+1)(y-3) = 0$ $y = \frac{1}{3} \text{ or } 3$ $x = 2(\frac{-1}{3}) - 2 \text{ or } x = 2(3) - 2$ $= -2\frac{2}{3} = 4$ | | |
| Linked | Solving Linear Equations / Nor | -Linear Graphs quad | lratic ar | nd cubic / Solving (| Quadra | tic Equations / Linea | r Graphs | | |

Knowledge Maps