## Toynhee Curriculum Knowledge Maps

## MATHS

## [Algebra]

## Sequences

## Keywords: Arithmetic / nth term / Geometric / Term / Quadratic / Iterate

| Definition / Description: | Arithmetic - a sequence where terms are found by adding or subtracting an equal amount. | Nth term - The general rule of a number sequence. | Geometric - A sequence in which you find each term by multiplying the previous term by a fixed value. | Term - a part of an equation, expression or sequence. | Quadratic - A sequence where the difference increases or decrease by an equal amount | Iterate - a quantity arrived at by iteration. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |


| Knowledge points: | Nth term of a linear sequence | Finding terms in a sequence | Nth term of a Quadratic sequence | Geometric Progression | Sequences by iteration |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Knowledge point examples: |  <br> The nth term of a linear sequence is always of the form $\mathrm{An} \pm \mathrm{b}$, where: <br> $A$, is the difference between each term and the next term. $b$ is the difference between the first term and $A$. $\begin{array}{lllll} \mathrm{n}: \underbrace{13}_{\underline{13-\frac{2 n}{2 n}}} \underbrace{11}_{-2} & \underbrace{2}_{-2} & \underbrace{3}_{-2} & 4 \\ \underbrace{9} \end{array}$ <br> In a descending sequence we find the zero term to discover what we are taking $A n$ way from. | From the sequence $5,12,19,26,33 \ldots$ work out the $50^{\text {th }}$ term. <br> The nth term of this sequence is $7 n-2$ <br> Find the 50th term by substituting $\mathrm{n}=50$ into the rule, $7 n-2$ $\begin{aligned} & =7 \times 50-2= \\ & 350-2=348 \end{aligned}$ | Find the nth term in the sequence: <br> $5,9,15,23 \ldots$ <br> The second differences are constant (2) so the sequence is quadratic and the coefficient of $n^{2}$ is 1 . So the nth term includes $1 n^{2}$. To find the remainder of the nth term, we subtract $1 n^{2}$ from our sequence and find the nth erm of the linear sequence left over: <br> $\begin{array}{llll}5 & 9 & 15 & 23\end{array}$ <br> $14 \quad 9 \quad 16$ <br> 4 5 6 $7 \ldots$ The nth term of this sequence is $\mathrm{n}+3$. <br> Nth term of quadratic sequence $=n^{2}+n+3$ | Geometric progression is a sequence of nonzero numbers where each term after the first is found by multiplying the previous one by a number. <br> Find the next two terms of the sequence $3 \underbrace{6}_{x^{2}} \underbrace{12}_{x^{2}} \underbrace{24}_{x^{2}} \ldots$ <br> The term to term rule here is $x 2$ therefore the next two terms are $\begin{aligned} & 24 \times 2=\underline{48} \\ & 48 \times 2=\underline{96} \end{aligned}$ | Find the first four iterations of the iterative formula $\begin{aligned} & x_{n+1}=3 x_{n}-2 \text { with } \\ & x_{1}=2 . \\ & x_{2}=3 x_{1}-2 \\ & =3 \times 2-2=4 \\ & x_{3}=3 x_{2}-2 \\ & =3 \times 4-2=10 \\ & x_{4}=3 x_{3}-2 \\ & =3 \times 10-2=28 \\ & x_{5}=3 x_{4}-2 \\ & =3 \times 28-2=82 \end{aligned}$ |

## Linked Notation and manipulation / Functions / Multiples, Primes, Factors / Index Numbers Knowledqe

| Keywords: | Expression / Simplify / Term / Variable / Substitute / Coefficient / Equivalent / Solve / Expand / Factorise |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Definition / Description: | Expression: Simplify: <br> an algebraic collect <br> statement terms | Term: part of an expression | Variable: a Subst <br> quantity exchan <br> that can or rep <br> have  <br> different  <br> values  | Coefficient: a number or letter multiplying a term | Solve: find the result | out kets | Factorise: separate into factor |
| Knowledge points: | Use and interpret notation Use letter symbols to represent unknown numbers in equations | Substitute into formulae Swap letter symbols in formulae for numbers to solve for an unknown | Algebraic vocabulary Understand and use expressions, equations, formulae, identities, inequalities and terms | Simplify expressions Collect like terms, expanding brackets, factorising into brackets | Rearrange formulae Balance terms about the equation sign to make another unknown the subject of the formula | Equiv identi Use prove expre use th symb | lence and es gebra to equivalent sions and identity |
| Knowledge point examples: | $a b$ in place of $a \times b$ <br> $3 y$ in place of $y+y+y$ and $3 x y$ <br> $a^{2}$ in place of $a \times a, a^{3}$ in place of $a \times a \times a$ <br> $\frac{a}{b}$ in place of $a \div b$ | Find the value of $x^{2}$ when $x=5$ <br> When $x=5$, replace the $x$ in $x^{2}$ with 5 to make (5) ${ }^{2}$. $5^{2}=25$ | Expression: $a+1$ <br> Equation: $b=a+1$ <br> Formula: $F=m a$ <br> Identity: $2+b \equiv b+2$ <br> Inequality: $a>1$ <br> Term: $a$ or $2 b$ | Simplify: $\begin{aligned} & a+b+2 a-2 b \\ & =a+2 a+b-2 b= \\ & 3 a-b \end{aligned}$ <br> Expand: $\begin{aligned} & 3(x+5) \\ & =(3 \times x)+(3 \times 5) \\ & =3 x+15 \end{aligned}$ <br> Factorise: $6 c-8 d$ $=2 \times 3 c-2 \times 4 d$ $=2(3 c-4 d)$ | Rearrange: $y=2 x$ +3 to make $x$ the subject $y=2 x+3$ <br> (-3 on both sides) $y-3=2 x$ <br> ( $\div 2$ on both sides) $\frac{y-3}{2}=x$ | $\begin{aligned} & b x \\ & 2(a \\ & 5 x+ \end{aligned}$ | $\begin{aligned} & b \equiv b^{3} \\ & \equiv 2 a+2 \\ & x \equiv 11 x \end{aligned}$ |

## Linked Knowledge Maps

## AXES AND CO-ORDINATES

## Keywords: Axis / Co-ordinate / Parallel / perpendicular / Gradient / Linear Graph / Reciprocal

## Definition / Descriptio

 n:
## Knowledge points:

## Knowledge point examples:

## Linked

Knowledge Maps

Axis: the axes are the reference lines that form the coordinate plane

Co-ordinates and coordinate grid

Co-ordinates: 2 Parallel: lines numbers that that never locate a specific point on a coordinate plane

## Plotting graphs from a table

 A linear graph can be draw by substituting values into a table,Perpendicular: Gradient: The Two lines at right angles to one another

## Graphs parallel to the $y$-axis

 A graph parallel to the $y$-axis will always be of the form $\mathrm{x}=\mathrm{c}$

Linear graph:
A visual representation of a straight line.

Reciprocal: What you multiply a number by to make 1.

## Graphs parallel to the $x$-axis

A graph parallel to the $x$-axis will always be of the form $y=c$

$(3,5)$
$x$-co ordinate $\quad y$-co ordinate


Functions / Non-Linear Graphs quadratic and cubic / Non-Linear Graphs other / Solving Linear Equations / Inequalities / Simultaneous equations / Sequences / Transformations / Linear Graphs / Linear Graphs - parallel and perpendicular lines

## Keywords:

Functions, input, output, inverse function, composite function, flow charts

## Definition /

 Description:
## Knowledge

 points:
## Knowledge point

 examples:
## Linked

 Knowledge MapsA function is a relationship between variables. The inverse function is the reverse process. A composite function is the succession of two functions.

Interpret simple functions as expressions with inputs and outputs


Write down the output $y$ as an expression in terms of $\boldsymbol{x}$. $y=5 x-8$


Work out the output when the input is 10
$(10-4) \div 2=3$

Understand and use the function notation

Given that $\mathrm{f}(x)=4 x-5$ work out
(a) $\mathrm{f}(-6)$
(b) $\mathrm{f}(0.5)$
(a) $4 x-6-5=-29$
(b) $4 \times 0.5-5=-3$

$$
\begin{aligned}
& 3 x+2=0 \\
& 3 x=-2 \\
& x=-2 / 3
\end{aligned}
$$

$$
g(x)=\frac{7 x-1}{2}
$$

$$
\mathrm{f}(x)=5 x+1, \mathrm{~g}(x)=x^{2}
$$

$$
\begin{array}{ll}
\text { Find } g^{-1}(x) & \mathrm{fg}(x)=f \\
y=\frac{7 x-1}{2} & =5 x^{2}+1
\end{array}
$$

$$
2 y=7 x-1
$$

Interpret and use the composite function the inverse function

$$
\begin{aligned}
& f(x)=3 x+2 \\
& \text { Solve } f(x)=0
\end{aligned}
$$

$$
2 y+1=7 x
$$

$$
x=\frac{2 y+1}{7}
$$

$$
g^{-1}(x)=\frac{2 x+1}{7}
$$

Solving linear equations

## INEQUALITIES

Keywords: Inequality, region, solve, equation, variable, linear, quadratic
Definition / An inequality is a statement showing two quantities that are not equal. They can be represented on a number line and on a Description: graph.

| Knowledge | Inequality notation <br> Know correct conventions of | Represent Inequalities on <br> a number line |
| :--- | :--- | :--- |

open circle for strict inequality and closed circle for inclusive inequality
$x$ is less than 5

$$
x \geqslant 2
$$

$x$ is greater or equal to 2

$$
x \leqslant 0
$$

$x$ is less than or equal to 0

$$
-3 \leqslant x<5
$$

$x$ is greater or equal to negative 3 , and smaller than 5

## $x>1$

```
\(x\) is greater than 1
\[
x<5
\]
Knowledge
point
examples:
```


## Solving linear inequalities

Solve inequalities in one and represent solution set on a number line and using set notation.

When we represent (plot) inequalities, we must show whether they include or exclude the starting number.
 a number line
Show inequalities on a number line using correct notation


## Graphical Inequalities

Represent inequalities on a coordinate grid


Shade the region on the graph that satisfies the two inequalities: $y \geq x$ and $x<2$


## Solve Quadratic inequalities

Solve quadratics and represent answers on a number line and on a graph

$$
x^{2} \leqslant 9
$$

Form \& solve an equation to find the two bounds.

$$
\begin{aligned}
& x^{2}=9 \\
& x=3 \text { or } x=-3 \\
& -3 \leqslant x \leqslant 3
\end{aligned}
$$

Solve the inequality

$$
x^{2}+3 x-4<0
$$

1. Factorise
2. Set $y=0$
3. Sketch function
4. It is $<0$ so we shade in under the x axis.

$-4<x<1$

## Linked Knowledge

Solving linear equations
Solving quadratic equations
Linear graphs
Non linear aranhe incluidinc nuadratie

## SOLVING EQUATIONS

| Keywords: | Solve / Equation / Coefficient / Inverse / Equal / Linear Equation |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Definition / <br> Description: | Solve: To find the answer/value of something | Equation: A mathematical statement that shows two things are equal |  | nt: A <br> letter <br> an <br> term | Inverse: The reverse or opposite | Equal: An equals sign shows the equality between two expressions | Linear Equation: An equation where no variable has a power greater than one |
| Knowledge points: | Solve simple linear equations by using inverse operations <br> Solve simple linear equations with integer coefficients where the unknown appears on one or both sides of the equation or where the equation involves brackets <br> Set up a simple linear equation to solve problems |  |  |  |  |  |  |
| Knowledge point examples: | $\begin{gathered} \text { One Step } \\ 3 \mathrm{a}=15 \\ (\div 3) \quad \begin{array}{l} (\div 3) \\ a=5 \end{array} \end{gathered}$ | $\begin{gathered} \text { Two Ste } \\ 4 \mathrm{a}-3= \\ (+3) \\ 4 \mathrm{a}=2( \\ (\div 4) \quad \\ a=5 \end{gathered}$ |  | $\begin{aligned} & \text { Witt } \\ & 2(\mathrm{a} \\ & 2 \mathrm{a} \\ & (-6) \\ & { }^{(-6)} \\ & (\div 2) \end{aligned}$ | Brackets $\begin{aligned} & 3)=11 \\ & 6=11 \\ & =5^{(-6)} \\ & =2.5^{(\div 2)} \end{aligned}$ | Unknowns on both sides $\begin{gathered} 3 a+7=5 a+11 \\ (-3 a) \\ 7=2 a+11 \\ (-11) \\ -4=2 a \\ (-11) \\ (\div 2) \\ \\ \\ (-3 a) \end{gathered}$ | Forming and solving equations: <br> Jack is y years old. <br> His brother John is 5 <br> years older than him. <br> The sum of their ages <br> is 21 . How old is <br> Jack? $\begin{gather*} y+y+5=21 \\ 2 y+5=21 \tag{-5} \end{gather*}$ <br> (-5) $\begin{aligned} & 2 y \end{aligned}=16$ <br> Jack is 8 years old |

## Linked Knowledge Maps:

Algebraic Manipulation and Notation / Linear Graphs / quadratic equations / inequalities / simultaneous equations / linear sequences

## Keywords: Axis / Co-ordinate / Parallel / perpendicular / Gradient / Linear Graph / Reciprocal

Definition / Descriptio n:

## Knowledge

 points:
## Knowledge

 point examples:
## Linked Knowledge Maps

## Co-ordinates: <br> 2 numbers that locate a never meet another

 specific point on a coordinate plane
## Parallel line:s

Lines that have the same gradient


## Finding the equation of a parallel line

Parallel line have the same gradient no matter the intercept


## Perpendicular lines

Perpendicular lines meet or join at right angles


Negative reciprocal
If if the gradients of two lines have a product of -1 they are perpendicular.

| Gradient <br> of the line | Gradient of <br> the <br> perpendicular <br> line |
| :---: | :---: |
| 3 | $-1 / 3$ |
| -2 | $1 / 2$ |
| $-1 / 4$ | 4 |
| $4 / 5$ | $-5 / 4$ |

## Gradient: The steepness of a line <br> Linear graph: A visual representation of a straight line.

Recipro What yo multiply number make 1.

Finding the equation of a perpendicular line
Find the gradient of the perpendicular line by finding negative reciprocal.


Find the equation of the line perpendicular to the line $y=1$ and passes through the points $(-4,4)$
Gradient of the $\quad \mathrm{m}=-2$
Sub in ( 3,5 ) into the $\quad y=-2 x+$
equation:
$4=-2(-4)+c$
$4=8+c$
$c=-4$

Functions / Non-Linear Graphs quadratic and cubic / Non-Linear Graphs other / Solving Linear Equations / Inequalities / Simultanec equations / Sequences / Transformations / Linear Graphs/ Axes and Coordinates

## Keywords: Axis / Co-ordinate / Parallel / perpendicular / Gradient / Linear Graph / Reciprocal

Definition /
Descriptio
$\mathrm{n}:$

## Knowledge

 points:
## Knowledge point examples:

Axis: the axes are the reference lines that form the coordinate plane

Co-ordinates: 2 numbers that locate a specific point on a coordinate plane

Parallel:
lines that never meet.

Perpendicular:
Two lines at right angles to one another

Gradient:
The
steepness of a line

Linear graph:
A visual representation of a straight line.

Reciproc What you multiply a number b) to make 1

## Intercepts

These are the points at which the line meets/crosses an axes


The equation Finding the equation of a of a line line from 2 points
$\mathrm{y}=\mathrm{mx}+\mathrm{c}$
$\mathrm{M}=$ gradient
C $=\mathrm{y}$-intercept
To begin find the gradient. Next substitute one coordinate into the equation to find the $y$-intercept
Find the equation of the line w passes through ( 3,6 ) and ( 2,2 Step 1 Calculating the gradien $\frac{\text { Diff in } y}{\text { Diff in } x}=\frac{6-2}{3-2}=\frac{3}{1}=3$
Step 2 Form the equation $y=3 x+c$
Step 3 Find the $y$-intercept
Substitute one of the co-ordinates into the equation $(3,6)$

$$
6=3(3)+c
$$

$$
c=-3
$$

Step 4 Complete the equation

$$
y=3 x-3
$$

## Linked <br> Knowledge Maps

Functions / Non-Linear Graphs quadratic and cubic / Non-Linear Graphs other / Solving Linear Equations / Inequalities / Simultanec equations / Sequences / Transformations Linear Graphs - parallel and perpendicular lines / Axes and Coordinates

## Non linear graphs

## Keywords: <br> Reciprocal / Asymptote / Exponential / Growth and decay / Radius / Non-Linear

Definition / Description:

## Knowledge points:

Knowledge point examples:

Use the table to plot the graph

$$
y=1 / x
$$

| $x$ | -4 | -3 | -2 | -1 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | $-\frac{1}{4}$ | $-\frac{1}{3}$ | $-\frac{1}{2}$ | -1 | 1 | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{4}$ |



## Linked Non-Linear Graphs quadratic and cubic / Linear Graphs / Functions /

Reciprocal: The Asymptote: the inverse of any distance between a number except 0 curve and a line which approaches but never touches zero

Reciprocal graph: remember the asymptote to the curve as we cannot divide by 0

Exponential: a function, where we use repeated multiplication on an initial value to get the output

Exponential Graph: An exponential graph in the form $y=a^{x}$ will cross the $y$ axis at the point $(0,1)$

Radius: The distance between the centre of a circle and it's circumference

Non-Linear: A graph which does not have a consistent gradient

Equation of a circle: The equation of a circle with the centre $(0,0)$ is expressed in the form: $x^{2}+y^{2}=r^{2}$

Use the table to plot the graph

$$
y=2^{x}
$$



| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 0.125 | 0.25 | 0.5 | 1 | 2 | 4 | 8 |



The following circles all have centre $(0,0$
Write down the equations of the circles.

$$
\begin{aligned}
\text { Radius } & =5 & & \text { Radius }=\frac{1}{8} \\
x^{2}+y^{2} & =25 & & x^{2}+y^{2}=\frac{1}{64}
\end{aligned}
$$

Non-linear Graphs - quadratic and cubic
Keywords: Quadratic / Parabola / Substitute / Cubic / Root / Solution

## Definition / Description: <br> Quadratic expression <br> contains terms up to and including $x^{2}$

## Knowledge points:

A Parabola is a curved graph formed from a quadratic equation. A parabola has a line of symmetry

## Substitute:

Exchange or replace variables
with amounts

Roots or solutions: of graphs are the values of $x$-coordinates of the poin where the graph crosses the $x$-axis

## Plotting graphs:

1. Complete a table of values for a graph, substituting different values of $x$ to find the $y$ coordinate
2. Join the points with a smooth line to create a curve

| Linked | Linear Graphs |
| :--- | :--- |
| Knowledge | Non-linear graphs - reciprocal, exponential and circle |
| Maps | Solving Quadratic equations |

## Cubic expression contains

 terms up to and including $x^{3}$3. Label the graph

## Knowledge point <br> Positive Quadratic in the form $y=x^{2}$

 examples:|  |
| :--- |
| Knowledge |
| point |



Negative quadratic in the Positive cubic graph $y=x^{3}$ form $y=-x^{2}$


Negative cubic graph $y=-x^{3}$


Complex cubic graph


## Solving Quadratic Equations

## Keywords: Quadratic Equation / Solution / Formula / Factorise / Discriminant

| Definition / Descriptio n: | Quadratic Equation: An equation where the maximum power is two | Solution: The answer to a quadratic equation | Formula: An equation to fund quantities when giv certain values | Factorise: To break up or to separate into factors | Discriminant: the part of the quadratic formula underneath the square root symbol |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Knowledge points: | Solving by factorising Use the product and sum | Solving when not equal to zero | Solving by completing the square | Solve using the quadratic formula - Use thew $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$ | Finding the amount of solutions a quadratic equation has. The value of the discriminant shows the amount of solutions a quadratic equation has |
| Knowledge point examples: | Factorise and solve $g^{2}+11 g+24=0$ <br> Factors of 24 <br> 1 and 24 $\begin{gathered} g^{2}+11 g+24 \equiv \\ (g+3)(g+8) \\ (g+3)(g+8)=0 \\ g=-3 \text { and } g=-8 \end{gathered}$ | A quadratic equation can only be solved when equal to zero <br> Factorise and solve: $\begin{array}{r} x^{2}+6 x+10= \\ -2 \\ x^{2}+6 x+8=0 \\ (x+4)(x+2)=0 \\ x=-4 \text { or }-2 \end{array}$ | Use $\left(x+\frac{b}{2}\right)^{2}-\left(\frac{b}{2}\right)^{2}+c$ <br> To complete the square before solving: $\begin{gathered} x^{2}+8 x+6=0 \\ (x+4)^{2}-10=0 \\ (x+4)^{2}=10 \\ (x+4)= \pm \sqrt{10} \\ x= \pm \sqrt{10}-4 \end{gathered}$ | Solve $3 x^{2}+8 x-5=0$ using the quadratic formula To 3 S.F $\quad a=3 b=8 c=(-5)$ $\begin{aligned} & x=\frac{-8 \pm \sqrt{(64-4 \times 3 \times-5)}}{2 \times 3} \\ & x=\frac{-8 \pm \sqrt{124}}{6} \\ & x=0.523 \text { or } x=-3.19 \end{aligned}$ | $b^{2}-4 a c>0$ <br> 2 solutions <br> $3 x^{2}-4 x-3=0$ $b^{2}-4 a c=$ <br> $(-4)^{2}-4 \times 3 \times(-3)=52$ <br> Two Solutions <br> $b^{2}-4 a c=0$ <br> 1 solution $\begin{gathered} 16 x^{2}+16 x+4=0 \\ b^{2}-4 a c= \end{gathered}$ <br> $16^{2}-4 \times 16 \times 4=0$ <br> $\frac{\text { One solution }}{b^{2}-4 a c<0}$ <br> $b^{2}-4 a c<0$ <br> No real solutions $\begin{gathered} 4 x^{2}+3 x+2 \\ b^{2}-4 a c= \end{gathered}$ $3^{2}-4 \times 4 \times 2=-23$ <br> No real solutions |

## Linked Knowledge Maps

Multiples, Primes, Factors / Notation and manipulation / Non-Linear Graphs quadratic and cubic / Solving Linear Equations /
Inequalities / Sequences / Simultaneous equations

SIMULTANEOUS EQUATIONS


## Linked Solving Linear Equations / Non-Linear Graphs quadratic and cubic / Solving Quadratic Equations / Linear Graphs Knowledge Maps

